

## Grade 12 LS – Physics

### Chapter 10 -A

### Capacitor with a L.F.G of square signal

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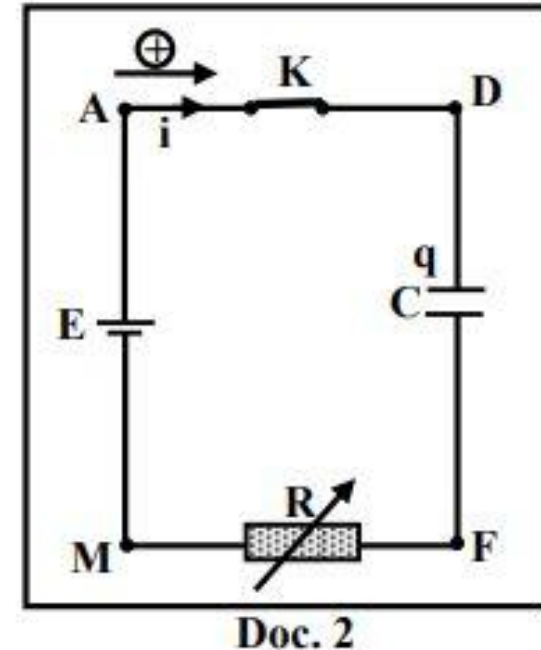
## Quiz

duration: 25 min



The aim of this exercise is to study the effect of the resistance of a resistor on the charging of a capacitor. For this aim we set-up the circuit of document 2 that includes:

An ideal generator of voltage  $u_{AM} = E$ ; a capacitor initially uncharged, of capacitance  $C = 4\mu F$ ; a resistor of adjustable resistance  $R$  and a switch  $K$ .



At the instant  $t_0 = 0$ , we close  $K$ , and the charging process starts.

## Theoretical study:

1. Derive the differential equation that describes the variation of the voltage  $u_{DF} = u_c$  during the charging of the capacitor
2. The solution of this differential equation has the form of:  $u_c = A + Be^{Dt}$ . Determine the constants A, B and D in terms of E, R and C.
3. Verify that the capacitor becomes practically fully charged at  $t = 5RC$ .
4. Indicate the effect of the resistance of the resistor on the duration of the charging of capacitor.

1. Derive the differential equation that describes the variation of the voltage  $u_{DF} = u_c$  during the charging of the capacitor.

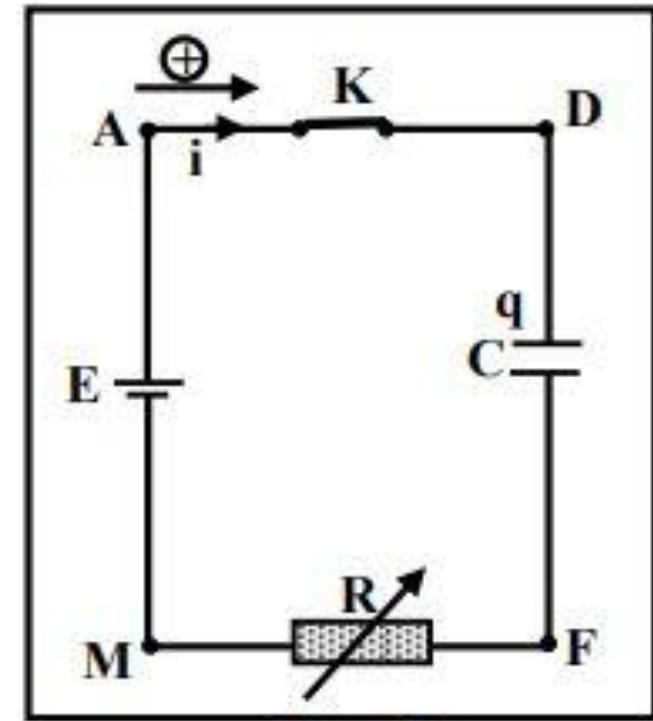
Using law of addition of voltages:

$$u_g = u_c + u_R \quad \Rightarrow \quad E = u_c + Ri$$

$$\text{But } i = \frac{dq}{dt} \text{ and } q = C \cdot u_c$$

$$i = C \cdot \frac{du_c}{dt}$$

$$E = u_c + RC \frac{du_c}{dt}$$



Doc. 2



2. The solution of this differential equation is  $u_c = A + Be^{Dt}$ .

Determine the constants A, B and D in terms of E, R and C.

$$u_c = A + Be^{Dt}$$

$$\frac{du_c}{dt} = B \cdot D \cdot e^{Dt}$$

$$E = A + Be^{Dt} + RC \cdot BD \cdot e^{Dt}$$

$$0 = -E + A + BDe^{Dt} + RC \cdot BD \cdot e^{Dt}$$

Substitute  $u_c$  and  $\frac{du_c}{dt}$  in differential equation:

$$0 = -E + A + BD \cdot e^{Dt} (1 + RC \cdot D)$$

$$-E + A = 0$$

$$E = u_c + RC \frac{du_c}{dt}$$

$$A = E$$

$$0 = -E + A + BD.e^{Dt}(1 + RC.D)$$



$$1 + RC.D = 0$$

$$1 = -RC.D \Rightarrow D = -\frac{1}{RC}$$

$$u_C = A + B.e^{D.t}$$

$$u_C = E + B.e^{-\frac{1}{RC}t}$$

$$\text{At } t=0, u_C = 0$$

$$0 = E + B.e^{-\frac{0}{RC}}$$

$$0 = E + B.e^0$$

$$0 = E + B \Rightarrow B = -E$$

$$u_C = E - Ee^{-\frac{t}{RC}}$$



3. Verify that the capacitor becomes practically fully charged at  $t = 5RC$ .

$$u_C = E(1 - e^{-\frac{t}{RC}})$$


$$u_C = E(1 - 0.0067)$$

$$u_C = E(1 - e^{-\frac{5RC}{RC}})$$

$$u_C = 0.99E \cong E$$

$$u_C = E(1 - e^{-5})$$

**For  $t = 5RC$**

**the capacitor becomes  
practically fully charged**



4. Indicate the effect of the resistance of the resistor on the duration of the charging of capacitor.

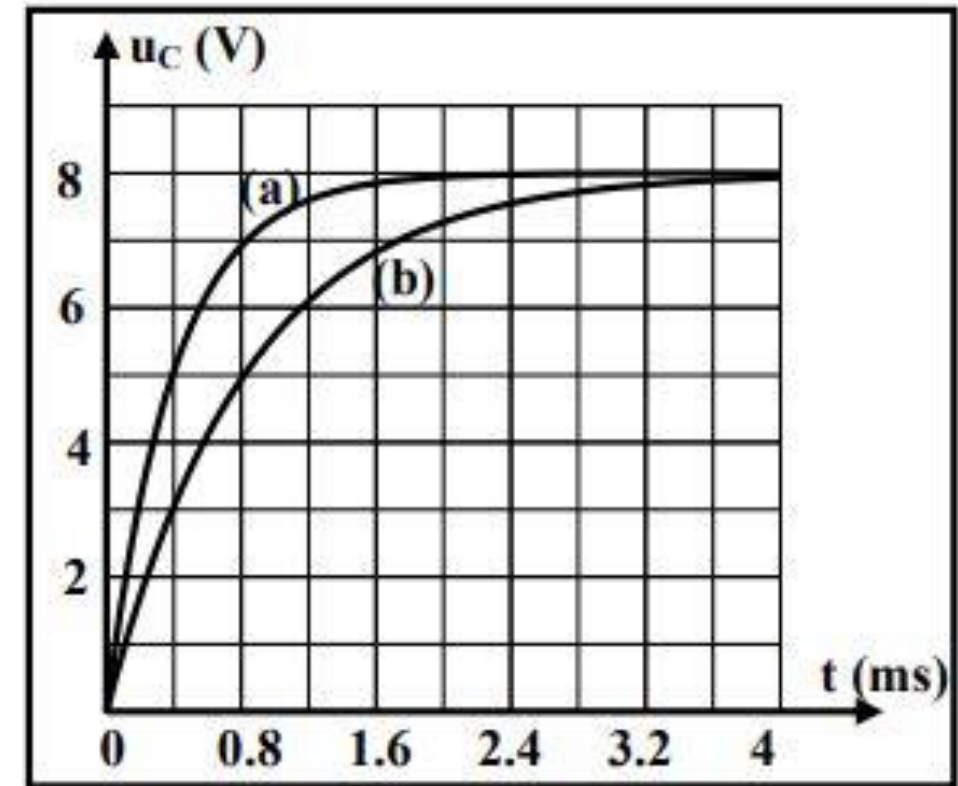
$$\tau = RC:$$

As the resistance increase, the charging time constant ( $\tau$ ) increases, therefore the charging process becomes slower.

## Experimental study:

We adjust  $R$  to two different values  $R_1$  and  $R_2$ ; an appropriate device allows to trace, for each value of  $R$ , the voltage  $u_C$  as a function of time (Doc.3):

curve (a) corresponds to  $R = R_1$  and  
curve (b) corresponds to  $R = R_2$ .



Doc. 3

# Quiz

duration: 25 min



1. Using the curves of document 3:

a. Specify the value of  $E$ .

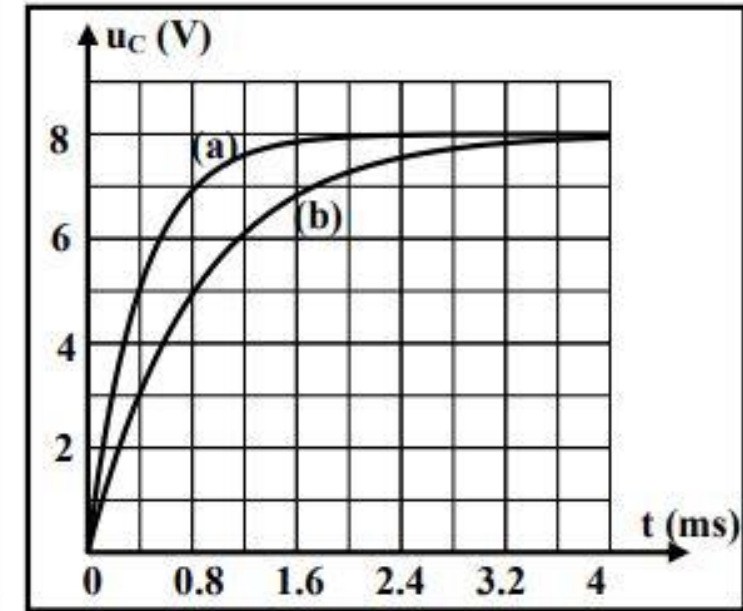
b. Specify, without calculation, whether that  $R_2$  is equal, greater, or less than the value of  $R_1$ .

c. Determine the values of  $R_1$  and  $R_2$ .

2. The capacitor is fully charged, the electric energy stored in the capacitor is  $W_C$ .

a. Is the value of  $W_C$  affected by the resistance of the resistor? Justify.

b. Deduce the value of  $W_C$ .



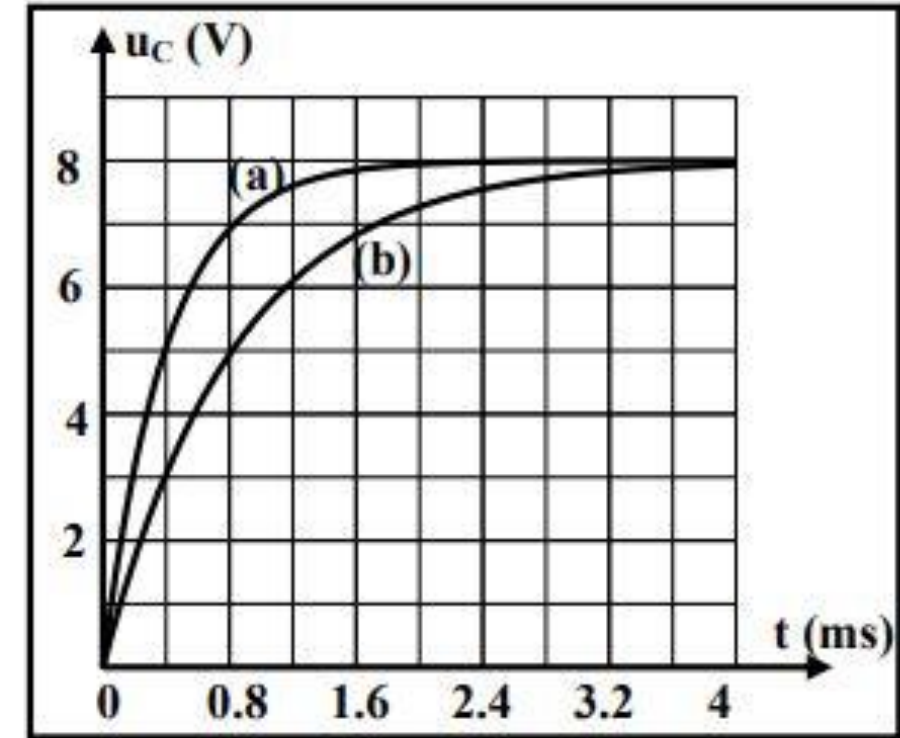
Doc. 3

1. Using the curves of document 3:  
a. Specify the value of  $E$ .

When the steady state is attained, the capacitor becomes fully charged.

Graphically, the steady state is attained at 8 V.

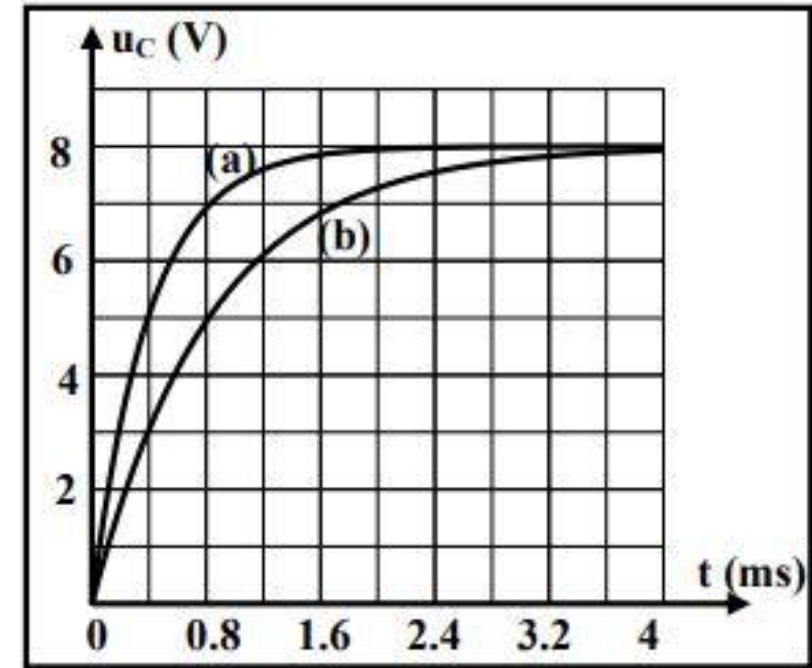
Therefore  $E = 8 \text{ V}$ .



Doc. 3

b. Specify, without calculation, whether that  $R_2$  is equal, greater, or less than the value of  $R_1$ .

- At any instant  $u_{C(a)}$  attains max or the capacitor is completely charged at  $t = 2s$ .
- At any instant  $u_{C(b)}$  attains max or the capacitor is completely charged at  $t = 3.36s$ .



Doc. 3

So, the charging process in curve (b) is slower, then  $R_2 > R_1$ .



c. Determine the values of  $R_1$  and  $R_2$ .

At  $t = \tau_1 = R_1 C$

$$u_{C(a)} = 0.63E = 0.63 \times 8 = 5V$$

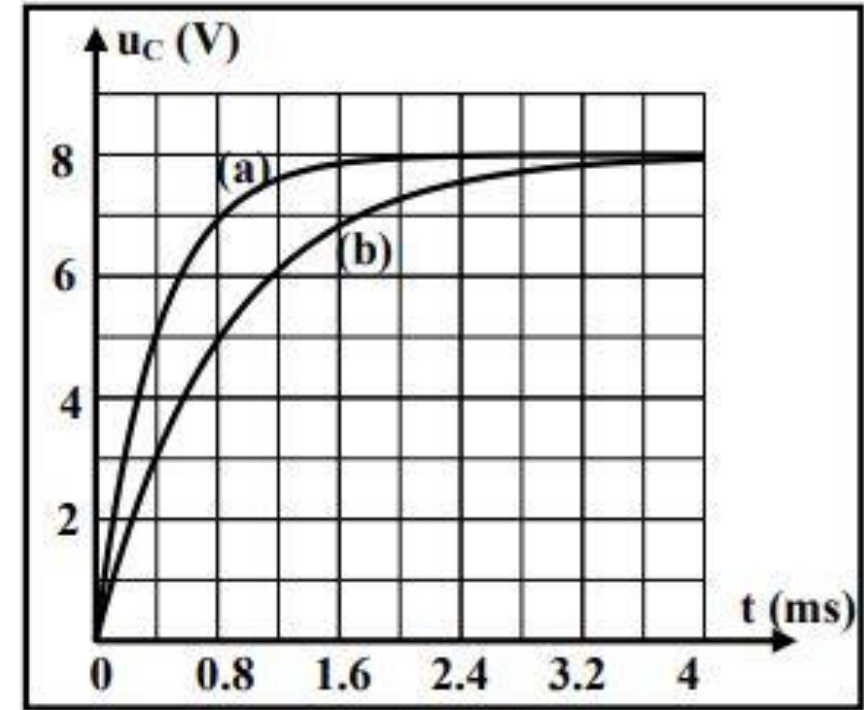
Then  $\tau_1 = 0.4ms$

$$\tau_1 = R_1 C \Rightarrow$$

$$R_1 = \frac{\tau_1}{C}$$

$$R_1 = \frac{0.4 \times 10^{-3}}{4 \times 10^{-6}} \Rightarrow$$

$$R_1 = 100\Omega$$



Doc. 3

At  $t = \tau_2 = R_2 C$

$$u_{C(b)} = 0.63E = 0.63 \times 8 = 5V$$

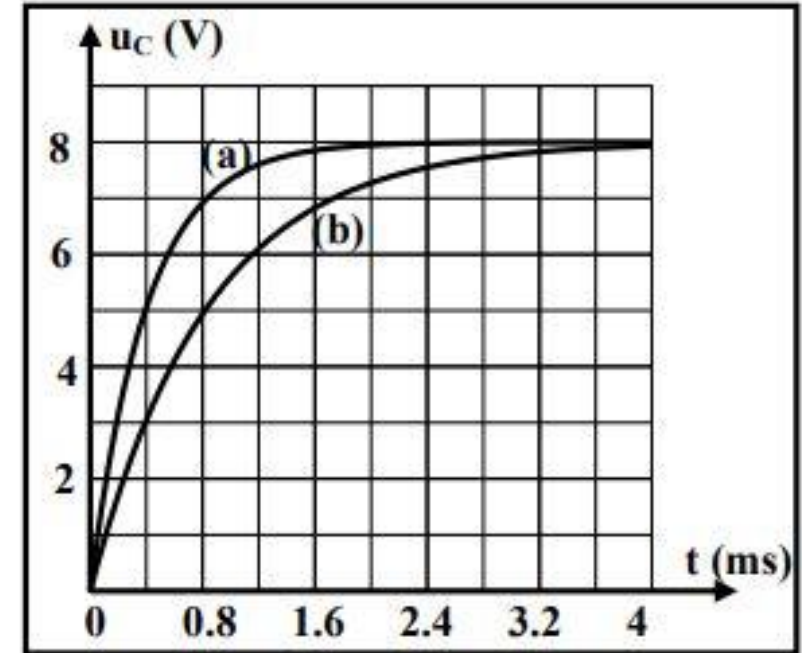
Then  $\tau_2 = 0.8ms$

$$\tau_2 = R_2 C \Rightarrow$$

$$R_2 = \frac{\tau_2}{C}$$

$$R_2 = \frac{0.8 \times 10^{-3}}{4 \times 10^{-6}} \Rightarrow$$

$$R_1 = 200\Omega$$



Doc. 3



2. The capacitor is fully charged, the electric energy stored in the capacitor is  $W_C$ .

a. Is the value of  $W_C$  affected by the resistance of the resistor? Justify.

$$W_C = \frac{1}{2} C \cdot u_C^2$$

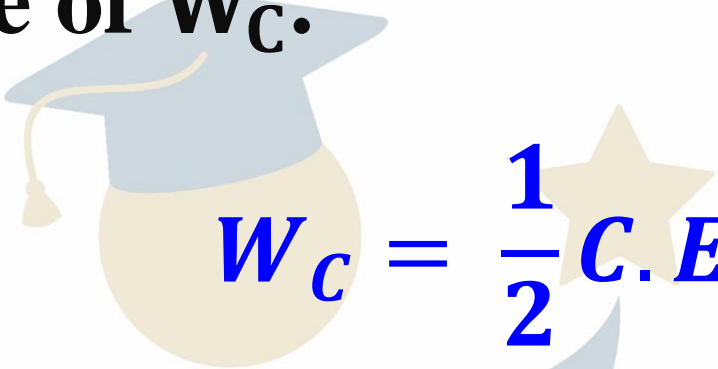
At the end of charging  $u_C = E$

$$W_C = \frac{1}{2} C \cdot E^2$$

Then the energy stored depends only on  $C$  and  $E$ .

Therefore, its value is **not affected** by the value the **resistance**.

b.Deduce the value of  $W_c$ .


$$W_c = \frac{1}{2} C \cdot E^2$$

$$W_c = 0.5 \times 4 \times 10^{-6} \times (8)^2$$

$$W_c = 1.28 \times 10^{-4} \text{ J}$$



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## Quiz 2

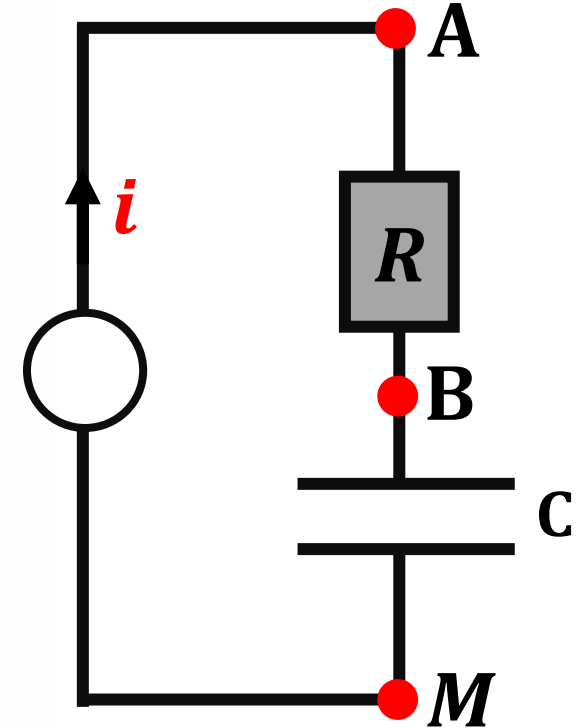
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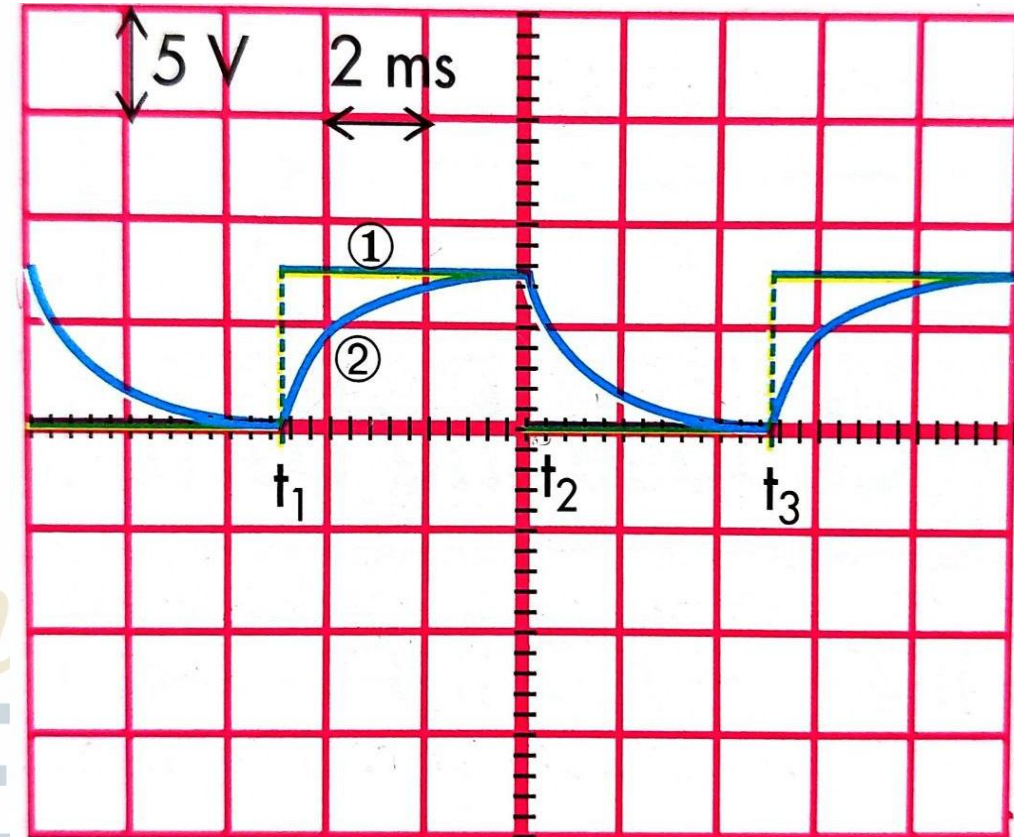
A (LFG) delivers a voltage  $(0, +E)$  of variable frequency  $f$ .

It feeds a circuit containing, in series, a resistor of resistance  $R = 10K\Omega$  and a capacitor of capacitance  $C = 100nF$  as shown in the figure.

The oscilloscope, conveniently connected, allows us to display the voltage  $u_{AM}$  across the generator on channel  $(Y_1)$  and the voltage  $u_{BM}$  across the resistor the capacitor on channel  $(Y_1)$ .



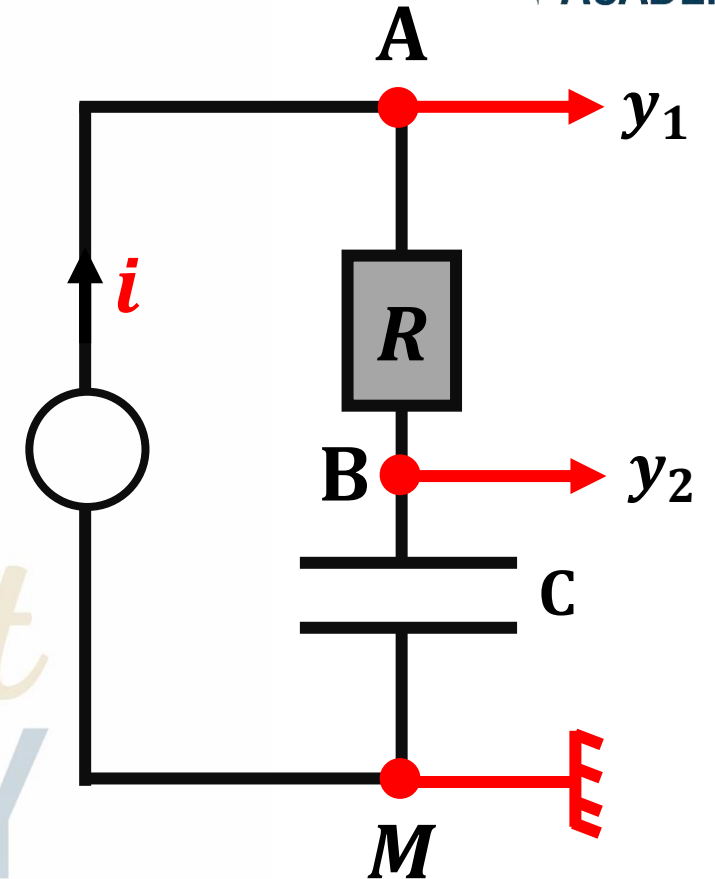
- 1) Showing the connections to the oscilloscope.
- 2) The waveform on the screen of the oscilloscope is represented by the adjacent figure. Identify the curve corresponding to voltage.
- 3) Determine the value of  $E$ .
- 4) Calculate  $f$ .



1) Showing the connections to the oscilloscope.

$u_{AM}$  displayed on channel ( $Y_1$ )

$u_{BM}$  displayed on channel ( $Y_2$ ).





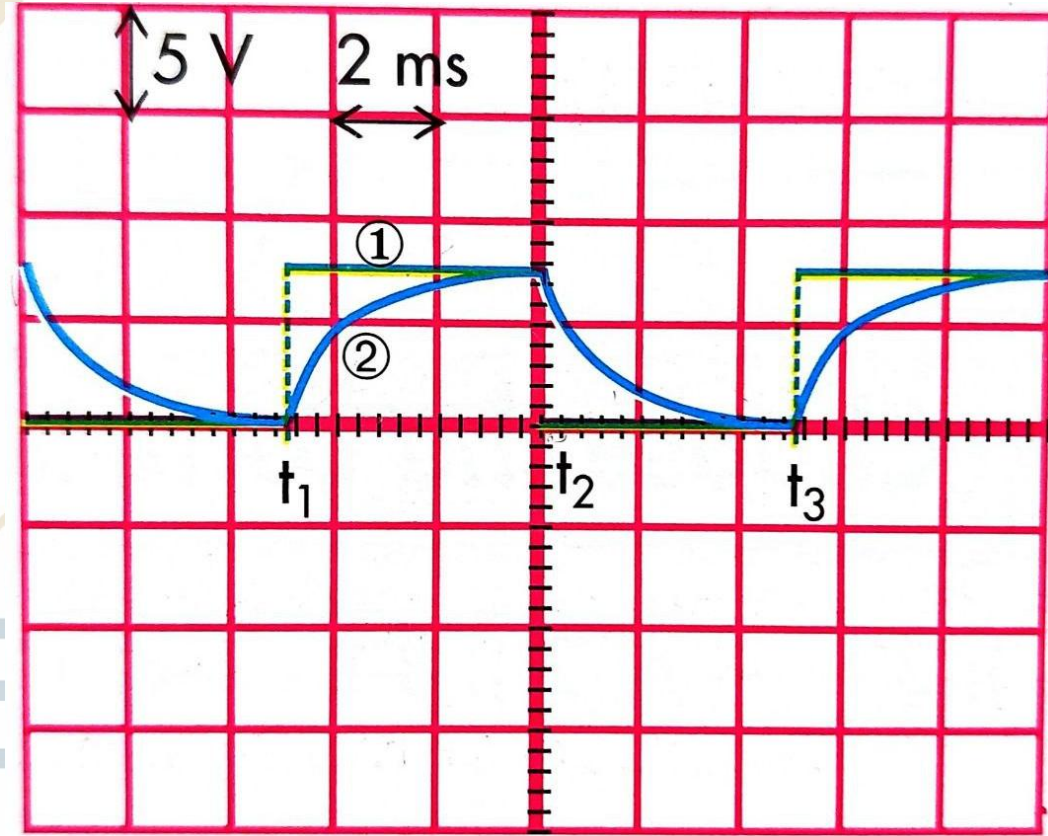
2) The waveform on the screen of the oscilloscope is represented by the adjacent figure. Identify the curve corresponding to voltage.

The channel ( $Y_1$ ) displays the voltage across the generator.

This voltage being  $(0; +E)$  then:

Curve 1 is for LGF

Curve 2 refers to capacitor





3) Determine the value of E.

$$u_{Cmax} = E = S_V \times y$$

$$u_{Cmax} = E = 5 \times 1.5$$

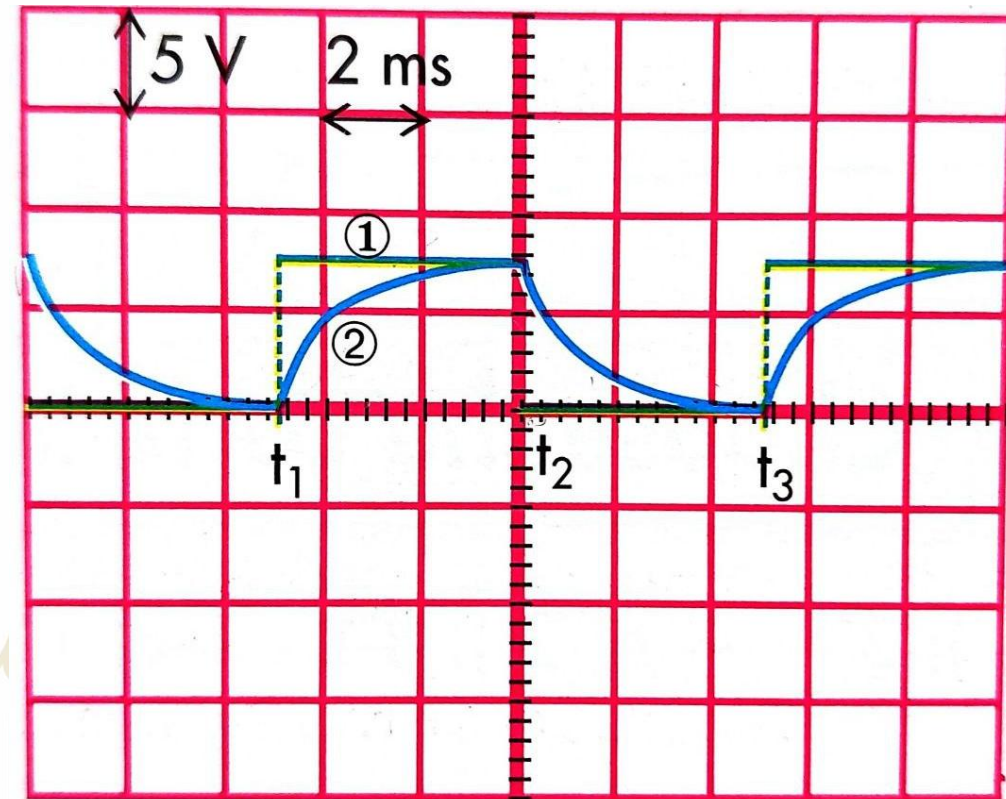
$$u_{Cmax} = E = 7.5V$$

4) Calculate f.

The period of the voltage delivered by the generator is

$$T = S_h \times x$$

$$T = 2ms \times 5div = 10ms \quad \left| \quad f = \frac{1}{T} = \frac{1}{(10 \div 1000)} = 0.01Hz \right.$$



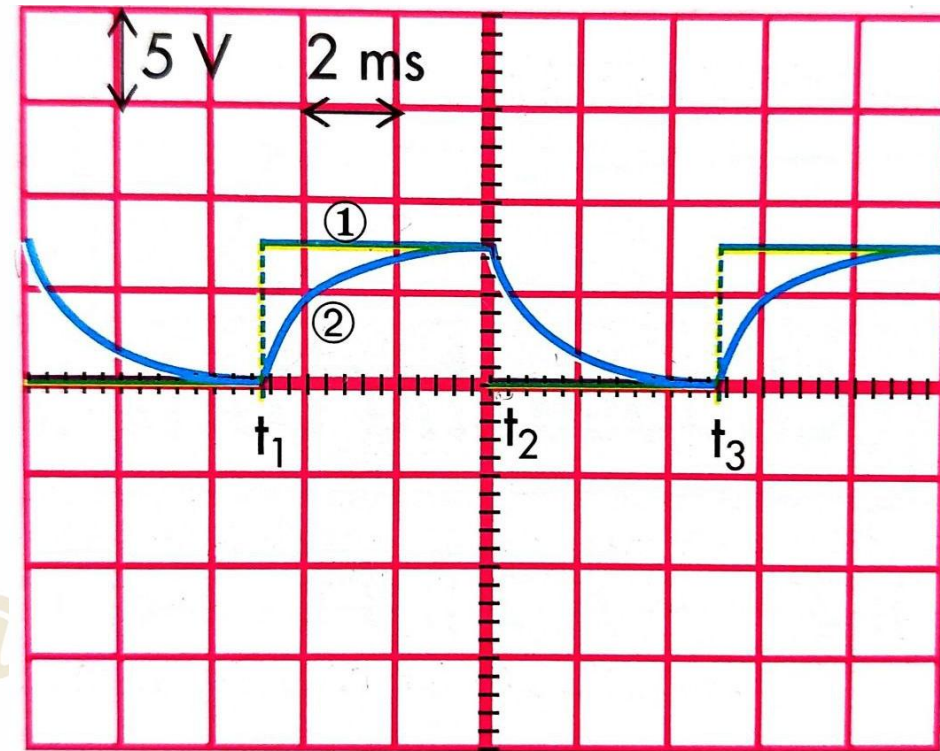
5) We are studying the waveform in the interval  $[0, t_1]$ .

a) Name the physical phenomenon that occurs in the circuit.

The voltage across the terminals of the capacitor decreases, then:  
Discharging of capacitor takes place.

b) What happens in the circuit during the interval  $[t_1; t_2]$

The voltage across the terminals of the capacitor decreases, then:  
charging of capacitor takes place.



$$R = 10K\Omega; C = 100nF$$

6) Calculate the time constant the compare it with T.



$$\tau = RC$$

$$\tau = 10000 \times (100 \times 10^{-9})$$

$$\tau = 0.001s = 1ms$$

$$T = 10ms$$

$$\tau = 1ms$$

$$T \gg \tau$$



